# Induction Continued

- prove $p(1)$ & $p(2)$ to prove it works for all even + odd integers $n \geq 1$

- $P(0), P(1)$

  if $p(k)$ then $P(k+1)$ and $p(k+2)$

Normal Induction: What we've been doing

Induction (strong form):

Let $P(n)$ be a sentence where $n$ is an integer.

① If $P(a), P(a+1), \ldots, P(b)$ where $a \leq b$, and
② For all integers $k \geq b$, if $P(m)$ for all (I.H.) integers $m$ value $a \leq m \leq k$ then $p(k+1)$
Then $P(n)$ for all integers $n \geq a$

Ex: The sequence $b_0, b_1, \ldots$ is defined by $b_0 = 12$, $b_1 = 29$ and for integers $k \geq 2$, $b_k = 5b_{k-1} - 6b_{k-2}$.

Prove that $b_n = 5 \times 3^n + 7 \times 2^n$ for all integers $n \geq 0$

Solution:

Base Cases $(n = 0, 1)$

$b_0 = 12 = 5+7 = 5 \times 1 + 7 \times 1 = 5 \times 3^0 + 7 \times 2^0$

$b_1 = 29 = 15 + 14 = 5 \times 3 + 7 \times 2 = 5 \times 3^1 + 7 \times 2^1$

Inductive Step: Suppose $k$ is an integer $k \geq 1$

Suppose $b_m = 5 \cdot 3^m + 7 \cdot 2^m$ for all integers $m$ (I.H)

Value $0 \leq m \leq k^*$ ← All values up to including $k$.

We prove that $b_{k+1} = 5 \cdot 3^{k+1} + 7 \cdot 2^{k+1}$

by the definition of the sequence

Now, $b_{k+1} = 5b_k - 6b_{k-1}$

by the I.H.

$$= 5(5 \cdot 3^k + 7 \cdot 2^k) - 6(5 \cdot 3^{k-1} + 7 \cdot 2^{k-1})$$
$$= 25 \cdot 3^k + 35 \cdot 2^k - 30 \cdot 3^{k-1} - 42 \cdot 2^{k-1}$$
$$= \underline{25 \cdot 3^k} + \underline{35 \cdot 2^k} - \underline{10 \cdot 3^k} - \underline{21 \cdot 2^k}$$
$$= 15 \cdot 3^k + 14 \cdot 2^k$$
$$= 5 \cdot 3 \cdot 3^k + 7 \cdot 2 \cdot 2^k$$
$$= 5 \cdot 3^{k+1} + 7 \cdot 2^{k+1}$$

Thus, $b_n = 5 \cdot 3^n + 7 \cdot 2^n$ for all integers $n \geq 0$

The sequence $b_0, b_1, b_2, \ldots$ is defined by $b_0 = 2$, $b_1 = 3$ and for integers $n \geq 2$,
$b_n = 3b_{n-1} - 2b_{n-2}$.

a) Compute $b_2, b_3, b_4, b_5$.

b) Guess a formula for $b_n$ where $n \geq 0$

c) Prove that your guess is correct of all integers $n \geq 0$

a) $b_2 = 5$ $b_3 = 9$ $b_4 = 17$ $b_5 = 33$

b) $b_n = 2^n + 1$ for all integers $n \geq 0$.